# Decision Support for Search and Rescue Response Planning

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## ABSTRACT

Planning, controlling and coordinating search and rescue operations is complex and time is crucial for survivors who must be found quickly. The search planning phase is especially important when the location of the incident is unknown. We propose, implement, solve, and evaluate mathematical models for the multiple rectangular search area problem. The objective is to define optimal or near-optimal feasible search areas for the available search and rescue units that maximize the probability of success. We compare our new model to an existing model on problem instances of realistic size. Our results show that we are able to generate, in a reasonable time, near optimal operationally feasible plans for searches conducted in vast open spaces. In an operational context, this research can increase the chances of finding survivors. Ultimately, as our models get implemented in the Canadian Coast Guard search planning tool, this can translate into more lives being saved.

### Keywords

Search and Rescue response, search planning, optimization, mixed-integer linear program, multiple rectangular search area

## INTRODUCTION

The act of searching is an important part of many humanitarian operations such as Search and Rescue (SAR) and mine counter measures, and of many surveillance operations for the purpose of protecting individuals, the environment, resources or infrastructures. Search and Rescue Units (SRU) including aircraft and vessels, teams of searchers, autonomous unmanned vehicles, may search for survivors, land or underwater mines, environmental spills, illicit activities, or abnormal behaviors. But how and where to search? The answer lies in good and efficient search planning that ensures the best use of scarce and constrained search resources. Search planning includes the definition of search areas and/or search paths in a way that maximizes the chances of an operation's success, often in degraded and rapidly changing conditions, in the presence of uncertainty on the whereabouts, the detectability, and on the conditions of the survivors, the threats, or the search objects.

In response to a distress incident, after having established plausible hypotheses regarding what might have happened and where, a SAR mission coordinator (SMC) must deal with the logistics of the search operations by allocating the available resources to search the established area of interest. The need for specific decision support systems that can assist a SMC in both the scenario and hypotheses building phase and the resource allocation phase has long been identified in Canada and elsewhere (Abi-Zeid and Frost 2005; Abi-Zeid, Nilo, Schvartz, et al. 2010; Aronica et al. 2010; Kratzke et al. 2010; Stone, Keller, et al. 2014; Małyszko and Wielgosz 2016; Bellantuono et al. 2016). Although both phases are of equal importance, we focus our research on resource allocation that mainly involves optimal search planning using search theory.

Search theory was one of the earliest Operations Research disciplines studied in the United States to address detection search problems (Stone, Royset, et al. 2016). As a matter of fact, it has been known, since the Second World War, that significant gains in search effectiveness are possible through the use of search theory. Frost and Stone 2001 presents many examples where search plans based on search theory did much better than less scientific methods. This was also demonstrated by Abi-Zeid and Frost 2005 where the use of a planning approach based on search theory was an improvement, in terms of the probability of success, over manual search planning methods. In addition, Ferguson 2008 reported that statistics have shown a significant increase in the number of lives preserved as a result of applying search theory concepts while more recently, Stone, Keller, et al. 2014 described how the application of search theory has helped locate the wreckage of AF 447. Search theory is also used in the area of autonomous searching by robots in structured environments, and by unmanned air vehicles for outdoor searching of large areas (Lau et al. 2008; Sato and Royset 2010; Kriheli et al. 2016; Venkatesan 2016; Bernardini et al. 2017).

A frequent practice in SAR response is to allocate search aircraft flying at the same altitudes to non-overlapping rectangular areas. This particular formulation of the search problem is known in the scientific literature as the Multiple Rectangular Search Areas (MRSA) problem (Discenza 1978; Abi-Zeid, Nilo, and Lamontagne 2011) where a grid of cells is superimposed over the search environment. A solution to the MRSA problem, also called a *search plan*, consists then of assigning SRUs to rectangles. A search plan is optimal if it maximizes the probability of finding lost search object(s), e.g., persons in water and/or missing vessels.

Very few papers have addressed the MRSA problem, a challenging optimization problem when many SRUs are present, when the area of interest is large, and when a fine grid with a small cell size is required. The literature contains various approaches including mixed-integer linear programming (MILP) (Discenza 1978) and constraint programming (CP) (Abi-Zeid, Nilo, and Lamontagne 2011). Discenza 1978 applied a heuristic to manage complexity by eliminating rows in the constraint matrix in order to obtain a matrix with the integral property (all the extreme points of the constraint polyhedron are integer valued). However, this reduces the set of feasible search plans and the optimal solution could be discarded by the reduction process. Furthermore, it does not take into account operational constraints. Richardson and Discenza 1980 proposed algorithms for the MRSA problem that could be applied to a maximum of five search units with cell sizes of 360 square Nautical Miles ($\text{NM}^2$). Abi-Zeid, Nilo, and Lamontagne 2011 designed and applied algorithms based on CP to obtain operationally feasible plans and used filtering heuristics to remove non-promising rectangles from the search space before using the CP solver. Their cell size of 25 $\text{NM}^2$ was more realistic.

In this paper, we revisit and reevaluate the performance of the MILP model proposed by (Discenza 1978) where rectangles are explicitly enumerated (MILP-Ex) and compare it against our proposed (MILP-Im) formulation based on an implicit definition of the rectangular search areas, implemented in two variants, with and without a startup heuristic. The work presented in this paper has served as a basis to obtain an ongoing contract to develop a search planning methodology to be integrated in an operational search planning system used in response to maritime SAR incidents in Canada.

This paper is structured as follows. We present search theory elements and the MRSA problem in the Background section. In the Methodology section we describe the (MILP) formulations. The following sections contain the experiments, results and discussion as well as the conclusion.

## BACKGROUND – SEARCH THEORY AND THE MRSA PROBLEM

Broadly speaking, available search effort is the quantification of the resources available for searching. In SAR operations, this is normally the available on-scene endurance of a SRU. Search effort is allocated to subareas of the search environment in order to maximize a figure of merit, such as the global probability of finding the search object(s), also called the probability of success (POS), subject to operational and physical constraints. In the MRSA problem formulation we consider, the allocated search effort, tracked individually for each SRU, is measured in distance units. Since a constant search speed is assumed for each SRU, this is equivalent to on-scene endurance. The MRSA problem normally deals with K SRUs. To each SRU $k \in \{1, \ldots, \text{K}\}$ corresponds a total effort $\text{E}_k$, in distance units, to be deployed over the search environment which is discretized by a grid consisting of cells.

Let $\mathcal{R}$ be the set of all rectangular search areas over a search grid of M rows and N columns for a total of

$$|\mathcal{R}| = \frac{\text{M}(\text{M}+1)\text{N}(\text{N}+1)}{4} \tag{1}$$

possible rectangular search areas. A search plan assigns each SRU to a single rectangular search area for a total of $O\left((|\mathcal{R}|)^{\text{K}}\right)$ possible search plans (feasible or not).


**(a) A single sensor track** **(b) A lateral range curve (fictional)**

**Figure 1. An illustration of the concept of lateral range curve**

Given a fixed amount of effort $\mathrm{E}_k$, many factors can influence the performance of SRU $k$ in rectangle $r$. In search theory, all these factors reduce to a single conditional probability of detecting a specific object type over a specific area under specific conditions given that the object is in the area searched.

In order to derive the conditional probability of detection, we introduce two core concepts: the lateral range curve and the sweep width (detectability index). The efficiency of a sensor can be characterized by a lateral range curve: Suppose that a sensor is traveling along a straight line (track) at a constant speed. The *lateral range curve* $p^{\mathrm{lrc}}(x)$ gives the probability that a search object is detected along a search track as a function of the distance $x$ at its closest point of approach. It is not a probability density function nor is it a cumulative density function. The underlying hypothesis is that the sensor's (searcher) track is infinitely long, in both directions (Figure 1). The area under the *lateral range curve* is called the sweep width. The larger the sweep width, the more efficient the sensor. The *sweep width* is defined as the integral of $p^{\mathrm{lrc}}(x)$ over $x$ on the interval $[-\infty, \infty]$, $\int_{-\infty}^{\infty} p^{\mathrm{lrc}}(x)\,\mathrm{d}x$.

A probability of containment $\mathrm{POC}_{ij}$, also called probability of whereabouts, is assumed to be known a priori for all cells $(i, j)$ of the grid. This is the probability that the search object is in cell $(i, j)$. The probability of containment of a rectangle $r \in \mathcal{R}$ is the sum of the cell probabilities that it contains:

$$\mathrm{rpoc}(r) = \sum_{(i,j)\in r} \mathrm{POC}_{ij} \tag{2}$$

An exponential detection function, that yields a lower bound on the probability of detection, is often assumed in optimization algorithms (Stone, Royset, et al. 2016). It is a function of the amount of effort $e$ deployed in rectangle $r$ and of the sweep width W of a sensor-search object combination under precise environmental conditions as follows:

$$\mathrm{rpod}(e, r) = 1 - \exp\left(\frac{-\mathrm{W} \cdot e}{|r| \cdot \mathrm{A}}\right) \tag{3}$$

where A is the area of a single cell and $|r|$ is the number of cells in rectangle $r$. A constant sweep width of W is assumed during the search, a realistic assumption for maritime SAR. It can be seen, from Eq. (3), that the detection probability increases with W. The same is true for the amount of effort $e$. It can also be seen that the probability of detection function follows the law of diminishing returns. The conditional probability of detection, as described above, does not take into account the probability of whereabouts. However, the probability of success, defined below, considers both probabilities.

The probability of success of a search unit inside a rectangular search area $r$ is the product of probability of the search object being located there (probability of containment) and that a detection occurs (conditional probability of detection). That is, given a rectangle $r \in \mathcal{R}$ and a SRU $k$, we have

$$\mathrm{rpos}(\mathrm{E}_k, r) = \mathrm{rpoc}(r) \cdot \mathrm{rpod}(\mathrm{E}_k, r) \tag{4}$$

where $\mathrm{E}_k$ is the effort deployed by SRU $k$ in rectangle $r$.

Let $\mathcal{A}$ be the set of all possible assignments of a SRU to a rectangular search area. The overall probability of success of the search plan $s \in \mathcal{A}^{\mathrm{K}}$ with K non-overlapping rectangles is the sum of the success probabilities local to each rectangular area. The objective of the MRSA problem is to maximize the probability of success of a search plan consisting of a set of SRU-rectangle assignments over the search grid as defined by Eq. (5).

$$\max_{\{r_1,\ldots,r_{\mathrm{K}}\}\in\mathcal{A}^{\mathrm{K}}} \sum_{k=1}^{\mathrm{K}} \mathrm{rpos}(\mathrm{E}_k, r_k) \tag{5}$$

subject to non-overlapping constraints. The formulation of this problem first appeared in (Discenza 1978).

### Operational and Physical Constraints

In practice, SAR mission coordinators aim at achieving an area coverage factor of around 1. This is the dimensionless ratio of the area effectively searched to the physical size of the area searched. Given a total amount of effort $E_k$ for SRU $k$, we define the coverage factor of $k$ in rectangle $r$ as:

$$\mathrm{cov}(r,k) = \frac{W \cdot E_k}{|r| \cdot A} \tag{6}$$

This is a measure of how thoroughly a search area is covered independently of any prior knowledge on the object's whereabouts. When a SRU follows a parallel-track search pattern, further constraints on the spacing of the parallel tracks can be added. Recalling that the search effort deployed by a SRU $k$, $E_k$, is the total track length in distance units and that the area of a rectangle $r$ is $|r| \cdot A$, we define the track spacing of SRU $k$ in rectangle $r$ as:

$$\mathrm{tracks}(r,k) = \frac{|r| \cdot A}{E_k} \tag{7}$$

By constraining the track spacing to be larger than some lower bound, we ensure that the turning radius is big enough for the search pattern to be feasible. These supplementary constraints, not found in (Discenza 1978), can be added to further restrict the set of feasible search plans $\mathcal{A}^K$ (Abi-Zeid, Nilo, Schvartz, et al. 2010).

## METHODOLOGY – SOLVING THE MRSA PROBLEM FOR SAR RESPONSE

In this section, we present two MILP formulations of the MRSA problem. The first formulation, MILP-Ex, requires an explicit enumeration of all the feasible rectangles in the search grid and is attributed to Discenza 1978. The second formulation, MILP-Im, is novel. It is based on the idea of using constraints to implicitly model rectangles and then letting the solver perform the enumeration of the rectangles under the assumption of a constant sweep width. Furthermore, we describe a myopic heuristic algorithm that can be used to provide a startup solution to the (MILP-Im) model.

### MILP with Explicit Enumeration of Rectangular Search Areas (MILP-Ex)

To build this model, we first compute the probability of success $\mathrm{rpos}(E_k, r)$ obtained by a SRU $k$ when it is assigned to rectangle $r$, for all rectangles $r \in \mathcal{R}$ and SRUs $k \in \{1, \ldots, K\}$ . We then define a set of binary variables to encode the assignment of SRU $k$ to a given rectangle $r$ as follows:

- $RECT_r^k$ equals 1 if SRU $k$ is assigned to rectangle $r$ in the search plan and to 0 otherwise.

This leads us to the following MILP:

$$\max_{RECT} \sum_{(r,k)\in\mathcal{A}} \mathrm{rpos}(E_k, r) \cdot RECT_r^k \tag{8}$$

subject to Constraints (9) to (10) where $RECT_r^k$ is a binary variable. Each cell of the grid is constrained to belong to a single assignment:

$$\sum_{(r,k)\in\mathcal{A}} RECT_r^k \cdot [(i,j) \in r] \leq 1 \qquad \forall (i,j) \in C \tag{9}$$

where $[(i,j) \in r]$ equals 1 if cell $(i,j)$ is in rectangle $r$ and 0 otherwise. Constraint (10) forces each SRU $k$ to be assigned to a single rectangle:

$$\sum_{r\in\mathcal{A}_k} RECT_r^k \leq 1 \qquad \forall k \in \{1, \ldots, K\} \tag{10}$$

$\mathcal{A}_k$ is the set of all rectangles such that rectangle $r$ is assigned to SRU $k$ in $\mathcal{A}$:

$$\mathcal{A}_k = \{r \mid (r,k) \in \mathcal{A}\} \tag{11}$$

The total number of feasible assignments in $\mathcal{A}_k$ can be reduced by applying the operational constraints from Eq. (6) and Eq. (7) leading to a simpler MILP.

**MILP with Implicit Enumeration of the Rectangular Search Areas (MILP-Im)**

We first describe the variables and the constraints needed to create $k$ non-overlapping rectangles. We then add the constraints required to compute the objective function.

*Constructing Non-Overlapping Rectangles*

We construct $k$ non-overlapping rectangles using Constraints 12 to 28. Consider the following variables encoding the membership of a cell to a rectangle, to a row, and to a column:

- $X_{ij}^k \in \{0, 1\}$ is equal to 1 if the cell $(i, j)$ belongs to rectangle $k$ and to 0 otherwise
- $ROW_i^k \in \{0, 1\}$ is equal to 1 if row $i$ belongs to rectangle $k$ and to 0 otherwise
- $COL_j^k \in \{0, 1\}$ is equal to 1 if column $j$ belongs to rectangle $k$ and to 0 otherwise

The following constraints ensure that a given cell $(i, j)$ belongs to rectangle $k$ if and only if row $i$ and column $j$ belong to rectangle $k$:

$$X_{ij}^k \geq ROW_i^k + COL_j^k - 1 \qquad \forall k \in \{1, \ldots, \mathrm{K}\}, i \in \{1, \ldots, \mathrm{M}\}, j \in \{1, \ldots, \mathrm{N}\} \quad (12)$$

$$X_{ij}^k \leq ROW_i^k \qquad \forall k \in \{1, \ldots, \mathrm{K}\}, i \in \{1, \ldots, \mathrm{M}\}, j \in \{1, \ldots, \mathrm{N}\} \quad (13)$$

$$X_{ij}^k \leq COL_j^k \qquad \forall k \in \{1, \ldots, \mathrm{K}\}, i \in \{1, \ldots, \mathrm{M}\}, j \in \{1, \ldots, \mathrm{N}\} \quad (14)$$

Note that given a SRU $k$ and a 4 by 5 search grid, and based on the above constraints, the following

$$X^k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

would be invalid. However, although they do not define rectangles, the following

$$X^k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (16)$$

and

$$X^k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad (17)$$

are valid. To avoid such situations, we introduce variables specifying the boundaries of a rectangle as follows:

- $\overline{ROW}_i^k \in \{0, 1\}$ is equal to 1 if the rectangle begins at a row $i'$ with $i' \leq i$ and to 0 otherwise
- $\underline{ROW}_i^k \in \{0, 1\}$ is equal to 1 if the rectangle ends at a row $i'$ with $i' \geq i$ and to 0 otherwise
- $\overline{COL}_j^k \in \{0, 1\}$ is equal to 1 if the rectangle begins at a column $j'$ with $j' \leq j$ and to 0 otherwise
- $\underline{COL}_j^k \in \{0, 1\}$ is equal to 1 if the rectangle ends at a column $j'$ with $j' \geq j$ and to 0 otherwise

We then add Constraints (18) and (19) to avoid skipping rows, and Constraints (20) and (21) to avoid skipping columns:

$$\overline{ROW}_i^k \leq \overline{ROW}_{i+1}^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, i \in \{1,\ldots,\mathrm{M}-1\} \quad (18)$$

$$\underline{ROW}_i^k \geq \underline{ROW}_{i+1}^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, i \in \{1,\ldots,\mathrm{M}-1\} \quad (19)$$

$$\overline{COL}_j^k \leq \overline{COL}_{j+1}^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, j \in \{1,\ldots,\mathrm{N}-1\} \quad (20)$$

$$\underline{COL}_j^k \geq \underline{COL}_{j+1}^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, j \in \{1,\ldots,\mathrm{N}-1\} \quad (21)$$

Let $\bar{i}$ and $\underline{i}$ correspond to the rows where rectangle $k$ begins and ends. The following constraints state that row $i$ belongs to a rectangle $k$ iff $\bar{i} \leq i \leq \underline{i}$:

$$ROW_i^k \geq \overline{ROW}_i^k + \underline{ROW}_i^k - 1 \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, i \in \{1,\ldots,\mathrm{M}\} \quad (22)$$

$$\overline{ROW}_i^k \geq ROW_i^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, i \in \{1,\ldots,\mathrm{M}\} \quad (23)$$

$$\underline{ROW}_i^k \geq ROW_i^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, r \in \{1,\ldots,\mathrm{M}\} \quad (24)$$

The following constraints are the same as above, but for columns.

$$COL_j^k \geq \overline{COL}_j^k + \underline{COL}_j^k - 1 \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, j \in \{1,\ldots,\mathrm{N}\} \quad (25)$$

$$\overline{COL}_j^k \geq COL_j^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, j \in \{1,\ldots,\mathrm{N}\} \quad (26)$$

$$\underline{COL}_j^k \geq COL_j^k \qquad \forall k \in \{1,\ldots,\mathrm{K}\}, j \in \{1,\ldots,\mathrm{N}\} \quad (27)$$

Finally, we need a set of $k$ non-overlapping rectangles as enforced by Constraint (28):

$$\sum_{k \in \{1,\ldots,\mathrm{K}\}} X_{ij}^k \leq 1 \qquad \forall i \in \{1,\ldots,\mathrm{M}\}, j \in \{1,\ldots,\mathrm{N}\} \quad (28)$$

### *The Objective function of MILP-Im*

The objective function of the MILP-Im model is built on the assumption that the sweep width is constant across the search grid. This assumption holds for maritime SAR when the environmental conditions (e.g., weather, crew fatigue) are constant during the search operations. Instead of enumerating, as with the MILP-Ex model, all the feasible SRU-rectangle assignments to compute their probability of success, we leave this task to the solver. We still need to compute the probability of detection prior to the solving process. However, since the sweep width is assumed constant, we only need to store it for all possible rectangle sizes in number of cells for each SRU. Therefore, instead of generating $O(|\mathcal{R}| \cdot \mathrm{K}) = O\left(\mathrm{M}^2\mathrm{N}^2\mathrm{K}\right)$ assignments and probability of success values, we store and generate $O(\mathrm{MNK})$ variables. Using $POS_{ij}^k \in [0.0, 1.0]$, the probability of success in cell $(i, j)$ of search grid $C$ for SRU $k$, we get the following objective function:

$$\max_{POS} \sum_{k=1}^{\mathrm{K}} \sum_{(i,j) \in C} POS_{ij}^k \quad (29)$$

In order to compute the value of the $POS$ variables, we define the following binary variables:

- $Y_a^k$ equals 1 if SRU $k$ is assigned to a rectangle of $a$ cells and to 0 otherwise.

We also add two supplementary binary variables sets to help in encoding the linear constraints of the $Y$ variables as follows:

- $U_a^k$ equals 1 if SRU $k$ is assigned to a rectangle with less than (or with exactly) $a$ cells and to 0 otherwise.
- $L_a^k$ equals 1 if SRU $k$ is assigned to a rectangle with more than (or with exactly) $a$ cells and to 0 otherwise.

Next, we add Constraints (30) to (33) where $\hat{\mathrm{M}}$ is a sufficiently large number, and where $\mathcal{R}^{\text{areas}}$ is the set of possible rectangle sizes $a$:

$$a - \sum_{(i,j)\in C} X_{ij}^k \le \hat{\mathrm{M}}\left(1 - U_a^k\right) \qquad \forall k \in \{1,\dots,\mathrm{K}\}, a \in \mathcal{R}^{\text{areas}} \quad (30)$$

$$\sum_{(i,j)\in C} X_{ij}^k - a \le \hat{\mathrm{M}}\left(1 - L_a^k\right) \qquad \forall k \in \{1,\dots,\mathrm{K}\}, a \in \mathcal{R}^{\text{areas}} \quad (31)$$

$$U_a^k + L_a^k = 2Y_a^k \qquad \forall k \in \{1,\dots,\mathrm{K}\}, a \in \mathcal{R}^{\text{areas}} \quad (32)$$

$$\sum_{a\in\mathcal{R}^{\text{areas}}} Y_a^k = 1 \qquad \forall k \in \{1,\dots,\mathrm{K}\} \quad (33)$$

We constrain the value of the $POS$ variables using the indicator variables of a rectangle size by Constraint (34):

$$POS_{ij}^k - \mathrm{POC}_{ij} \cdot \mathrm{pod}(\frac{1}{a}\mathrm{E}_k, (i,j)) \le \hat{\mathrm{M}}\left(1 - Y_a^k\right) \qquad \forall k \in \{1,\dots,\mathrm{K}\}, a \in \mathcal{R}^{\text{areas}}, (i,j) \in C \quad (34)$$

Furthermore, Constraint (35) forces the probability of success of a SRU $k$ in a cell $(i, j)$ to be null if $k$ is not located in $(i, j)$:

$$POS_{ij}^k \le X_{ij}^k \qquad \forall k \in \{1,\dots,\mathrm{K}\}, (i,j) \in C \quad (35)$$

Finally, the set $\mathcal{R}^{\text{areas}}$ is restricted using operational constraints described in Eq. (6) and Eq. (7).

### The MILP-Im with a Myopic Heuristic Based on the Probability of Success (MILP-ImH)

In order to jump-start the MILP-Im, we propose a simple myopic heuristic for the MRSA problem that first consists of enumerating all the rectangles and computing the probability of success of each SRU-rectangle assignment. Second, the SRU-rectangle assignment with the highest POS is chosen as part of the search plan. This heuristic can provide a good starting solution for the MILP-Im and is computed in polynomial time. We call the implementation of MILP-Im with this heuristic the MILP-ImH model.

## EXPERIMENTS

Since time is of the utmost importance when lives are at risk following a SAR incident, it is crucial that any decision support for operational search planning yield good search plans in a short time period. We therefore conducted experiments to evaluate and compare performances in terms of solution quality and execution time using realistic search areas. We conducted experiments using the two models presented above, i.e., the model with enumerated rectangles (Discenza 1978) with additional operational constraints and our novel formulation with an implicit enumeration of the rectangles. In total, we compared the performance of three variants of the models (MILP-Ex, MILP-Im, and MILP-ImH) in terms of the objective function value (probability of success) attained within 15 minutes, and of the time required to obtain that solution. The experimental framework was developed in C++ and built upon the IBM ILOG CPLEX solver version 12.6.1. The comparison was conducted on the following four grid instances:

- grid A: 13 by 17 cells, a total area of 5,525 $\mathrm{NM}^2$ (18,950 $\mathrm{km}^2$);
- grid B: 7 by 95 cells, a total area of 16,625 $\mathrm{NM}^2$ (57,022 $\mathrm{km}^2$);
- grid C: 30 by 30 cells, a total area of 22,500 $\mathrm{NM}^2$ (77,173 $\mathrm{km}^2$); and
- grid D: 47 by 49 cells, a total area of 57,575 $\mathrm{NM}^2$ (197,476 $\mathrm{km}^2$).

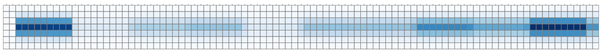


**Figure 2. The whereabouts of a search object on a grid of 7 by 95 cells (grid B)**

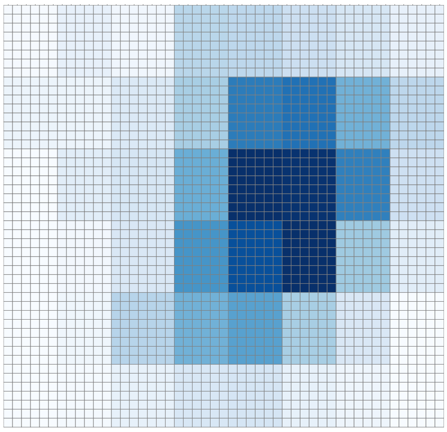


**Figure 3. The whereabouts of a search object on a grid of 47 by 49 cells (grid D)**

Each of the grids has a different distribution of the whereabouts of the search object (probability of containment). As an illustration, we present on Figures 2 and 3, the probability of containment of grids B and D respectively. A darker color cell has a higher probability of containment. For each grid, we generated two problem instances, one with 4 SRUs, and one with 5 SRUs. We applied the coverage constraint to reduce the solution space by enforcing a coverage such that $.5 \leq \mathrm{cov}(r, k) \leq 2.5$, for all rectangles $r$ and SRUs $k$. We also included track spacing constraints such that $.5$ NM $\leq \mathrm{tracks}(r, k) \leq 2.5$ NM for all rectangles $r$ and SRUs $k$ to guarantee feasible search plans. It should be noted that the above constraints are realistic operational constraints.

### Results and Discussion

The optimal value of the probability of success depends on the problem instance. To facilitate the comparison across instances of different sizes (both in terms of the number of SRUs involved and in terms the total number of cells in the grids), of different probabilities of containment and of different sweep widths, we present the percentage of improvement achieved by each incumbent solution. For each problem instance, the highest known objective value corresponds to a 100 % improvement. Figures 4 to 7 present the percentage of the best objective achieved versus the preparation and the solving time in seconds. The preparation time includes the time required to generate the model and to load it into memory. For the MILP-ImH, this also includes the time required to run the myopic heuristic that provides a starting solution. We chose to include the preparation time in the comparison since SAR operations are time critical. A decision support system based on a MILP solver for SAR planning needs to prepare the model first before solving it. In such a practical context, it is therefore important to identify the models that are more time-consuming to prepare.

By looking at the instances in an increasing order of complexity (from Figures 4 to 7), we notice that the MILP-Ex model scales poorly in comparison to the MILP-Im model. It is true, however, that the MILP-Ex model enables the solver to find high quality solutions much faster than the MILP-Im model on sufficiently small problem instances, e.g., on the instances of grid A. For the larger instances, i.e., the ones of grids C and D, the MILP-Ex model does not fit into memory which justifies the need for an alternative model as the size of the problem increases. The MILP-Im model consumes less memory and less time in the preparation phase. It thus has the advantage of allowing the solver to quickly start its search for solutions, which is critical for SAR response management since an SMC might need to stop the solver earlier to task the SRUs, or to try multiple scenarios which involves generating different MILP models. In an operational SAR setting, it is generally agreed that a first executable search plan should be found within 3 minutes. One problem that remains when using the MILP-Im model alone is that the first few solutions found by the solver might be low quality solutions, i.e., a low probability of success. By using a heuristic, we can provide a good starting point for the solver which is exactly what is done when using the MILP-ImH model.

Figure 8 shows the search plans obtained with 5 SRUs on grid B. Each rectangle corresponds to an area assigned to a SRU. The search plan of Figure 8a is the one returned by the myopic heuristic (the first solution of MILP-ImH). The search plans of Figures 8b, 8c and 8d are the best search plans found within 15 minutes (900 seconds) when

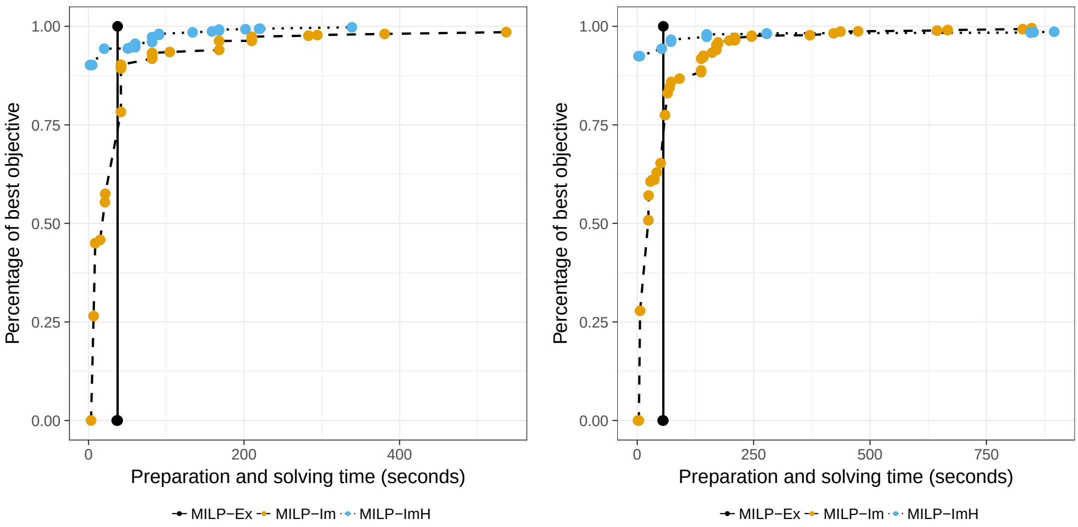



**Figure 4. Grid A: 13 by 17 cells with 4 SRUs (left) and 5 SRUs (right)**

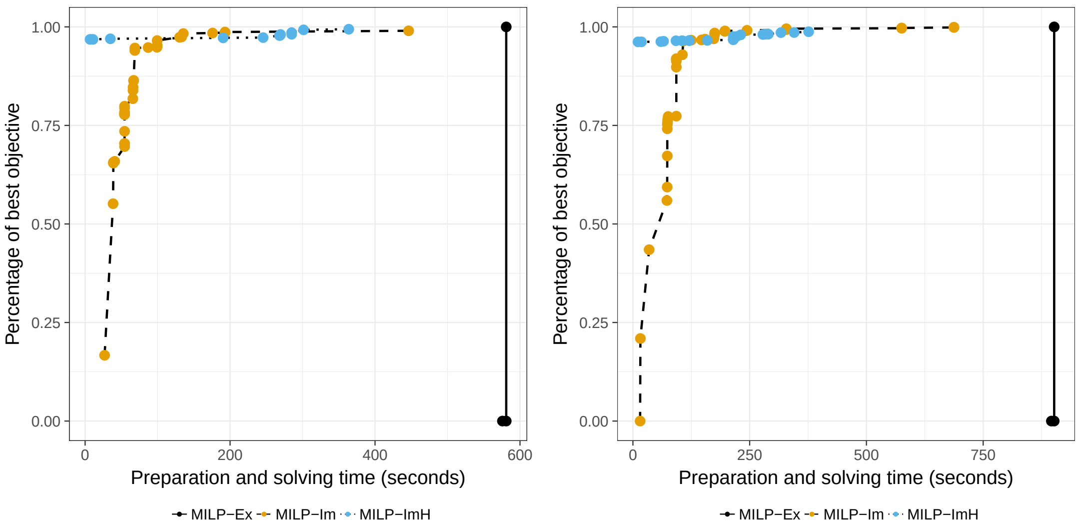



**Figure 5. Grid B: 7 by 95 cells with 4 SRUs (left) and 5 SRUs (right)**

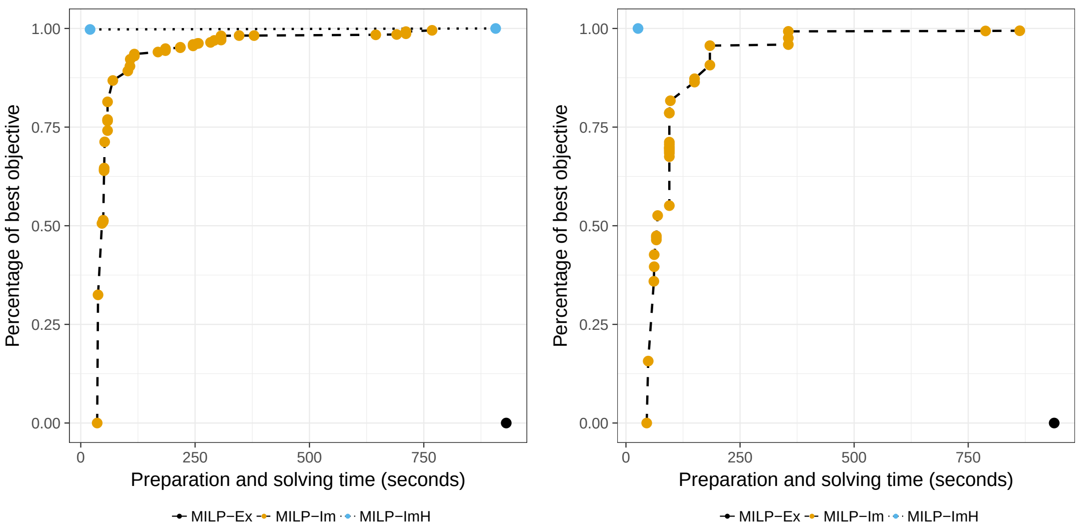



**Figure 6. Grid C: 30 by 30 cells with 4 SRUs (left) and 5 SRUs (right)**

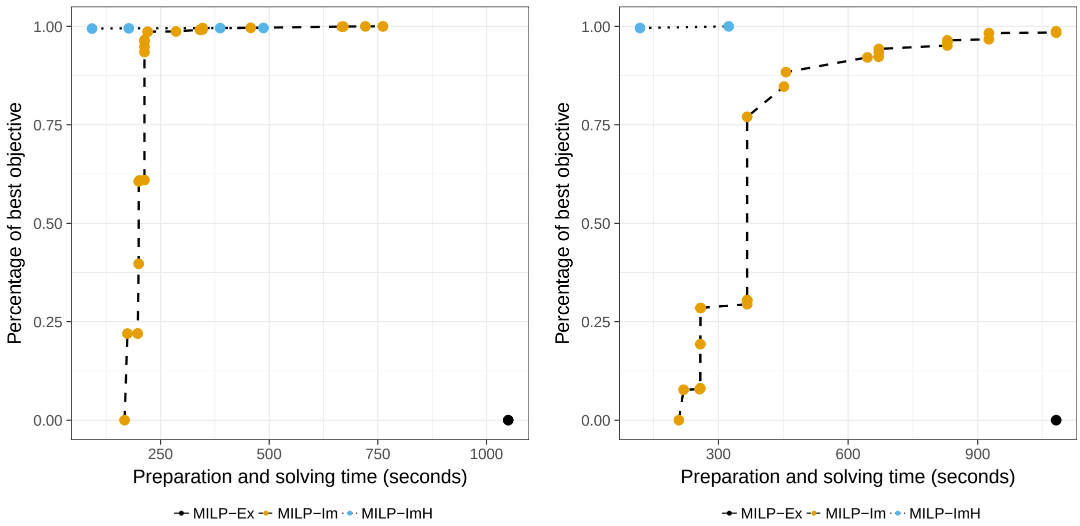
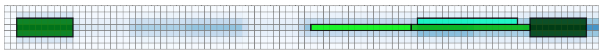
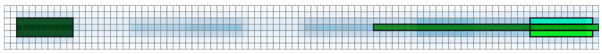
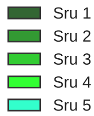



**Figure 7. Grid D: 47 by 49 cells with 4 SRUs (left) and 5 SRUs (right)**

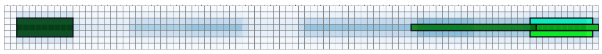


**(a) Myopic heuristic search plan,**
**96 % of improvement after 1 second**

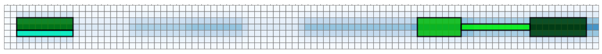





**(b) MILP-Ex search plan,**
**100 % of improvement after 896 seconds (including preparation time)**



**(c) MILP-Im search plan,**
**98 % of improvement after 684 seconds (including preparation time)**

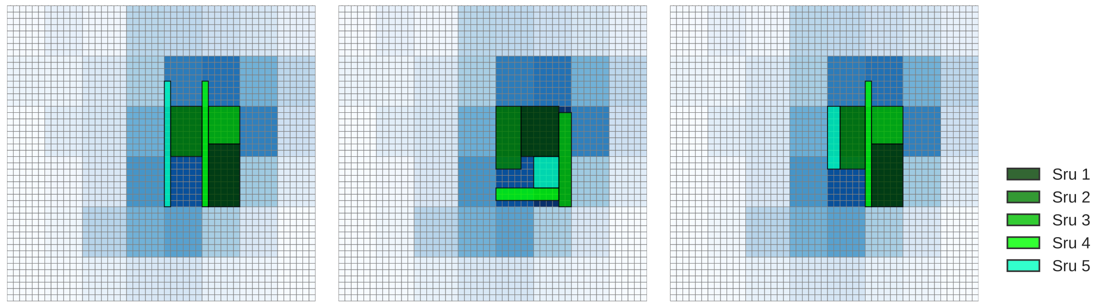


**(d) MILP-ImH search plan,**
**99 % of improvement 377 seconds (including preparation time)**

**Figure 8. Grid B: proposed search plans with 5 SRUs on a grid of 7 by 95 cells**

using the MILP-Ex model, the MILP-Im model and the MILP-ImH model respectively. The distribution of the whereabouts of the object in this particular instance has two important centers of mass. The MILP-Ex model found the optimal solution after almost 15 minutes. We note that the solution of the MILP-Im model is close, qualitatively, to this particular optimal solution. Moreover, on this problem instance, the myopic heuristic (the first solution of MILP-ImH) allowed the solver to obtain a high quality solution faster when using the MILP-ImH model. This particular solution was generated at least 8 minutes (480 seconds) earlier than the optimal solution and its objective value lies within 1 % of the optimal.

Figure 9 shows the search plans obtained with 5 SRUs on grid D. The search plan of Figure 9a is the one returned by the myopic heuristic (the first solution of MILP-ImH). The search plans of Figures 9b and 9c are the best search plans found within 15 minutes (900 seconds) by the MILP-Im model and the MILP-ImH model respectively. Note that the MILP-Ex model did not produce any search plan on that problem instance before the 15 minutes time limit. Following a close examination of these search plans, we see that the search plan proposed by the MILP-ImH model closely resembles the solution of the myopic heuristic (the first solution of MILP-ImH). Nonetheless, the search plan of the MILP-ImH model is still an improvement over the search plan proposed by the myopic heuristic. One particularity of this problem instance is the distribution of the whereabouts which has a circular shape centered on the last known position of the search object. The objective value of the best search plan found within 15 minutes, obtained after 324 seconds by MILP-ImH model corresponds to an improvement of 100 % since it is the best known




**(a) Myopic heuristic search plan, 99 % of improvement after 23 seconds**

**(b) MILP-Im search plan, 98 % of improvement after 990 seconds (including preparation time)**

**(c) MILP-ImH search plan, 100 % of improvement after 324 seconds (including preparation time)**

**Figure 9. Grid D: proposed search plans with 5 SRUs on a grid of 47 by 49 cells**

solution. Although the MILP-Im model achieved only an improvement of 98 % within 15 minutes, that particular solution was found after approximately 15 minutes (990 seconds). That is, the MILP-Im model found a solution within 2 % of the best-known solution after more than twice the time required (324 seconds) by the MILP-ImH model to find the best-known solution (100 %).

## CONCLUSION

In this paper, we have presented a novel model based on mixed-integer linear programming to solve the search planning problem of allocating multiple search resources in response to a Search and Rescue incident. The algorithms developed are meant to be implemented in a decision support system for search mission coordinators who must plan and coordinate SAR response when time is a critical factor. Our model proposes search plans that assign search aircraft to non-overlapping rectangular search areas in such a way to maximize the probability of success while taking into account operational constraints. We compared two versions of our model, with and without a startup heuristic, to an older existing model proposed by Discenza 1978, using problem instances of realistic size. We showed that we are able to generate, in a reasonable time, operationally feasible plans for searches conducted in open and vast spaces. One particularly interesting result is the ability of our myopic heuristic to quickly generate quite a good search plan as a first solution, albeit not an optimal one. This is valuable since it quickly provides the search mission coordinator with an initial plan that could be improved upon while resources are being tasked or other preparatory activities are conducted. The work presented here served as a basis for an ongoing contract to implement search planning optimization in a decision support tool. More experiments with more cases are planned to thoroughly evaluate performances and make final recommendations. Ultimately, as our models get implemented in the Canadian Coast Guard operational decision support tool, we can expect an increase in the chances of finding survivors more quickly and in more lives being saved.

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